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Transport in the Outer Core of Neutron Stars

CSQCD 2018, CUNY, New York

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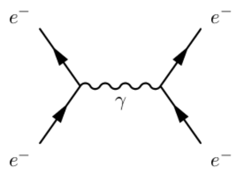
[S. Stetina, E. Rrapaj , S. Reddy, Phys.Rev. C97 (2018) no.4, 045801]
[S. Stetina, in preparation]

Phenomenological relevance



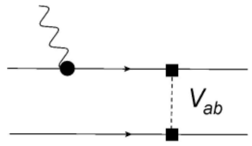
transport determined by

correlations of QED + strong int.

Photon spectrum:

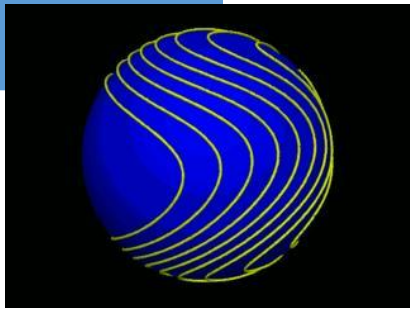
- collective modes
- Screening and Landau damping

Scattering rates of fermions



relevant for

- damping of hydro/R modes
- Neutron star spin evolution
- thermal relaxation
- (plasmon decay)

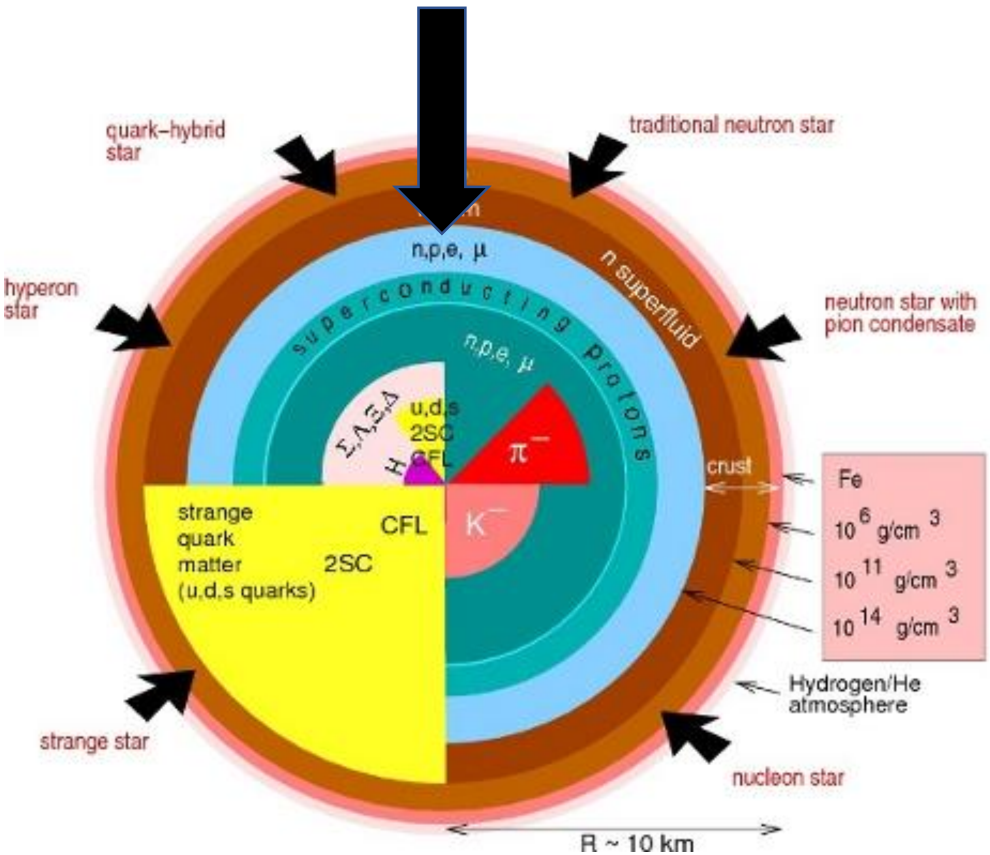


→ **electrons under NS conditions are relativistic, degenerate, weakly interacting**

→ **important contribution to transport**

The outer core of neutron stars

homogeneous plasma of electrons, muons, protons, and neutrons



[Weber, J. Phys. G27, 465 (2001)]

stable homogeneous nuclear matter

→ ß equilibrium and charge neutrality

$$\mu_n - \mu_p = \mu_e = \mu_\mu \ , \qquad n_e + n_\mu = n_p$$

→ degenerate QED plasma of electrons, muons and protons

→ protons and neutrons form strongly interacting Fermi liquid

critical densities

→ lower critical density (spinodal point)

$$n_c \sim 0.7\, n_0$$

→ onset of muons ($\mu_e = m_\mu$)

$$n_\mu \sim 0.75\, n_0 - 0.8\, n_0$$

Strongly interacting Fermi Liquid (I)

Landau energy functional:

[N. Chamel, P. Haensel, PRC.73, 045802]

$$E[n,\boldsymbol{j}] = \sum_{T=0,1} \delta_{T0}\frac{\hbar^2}{2m}\tau_T + C_T^n\{n_b\}\, n_T^2 + C_T^\tau\, n_T\, \tau_T + C_T^{\boldsymbol{j}}\, \boldsymbol{j}_T^2$$

Functional dependence on nucleon density *n*, kinetic energy density τ , and current density *j*

$$V_{ab} = \frac{\delta^2}{\delta n_a \delta n_b} E[n,\boldsymbol{j}] \ , \qquad\qquad (V_{ab})_{ij} = \left(\frac{\delta^2}{\delta \boldsymbol{j}_{a,i}\delta \boldsymbol{j}_{b,j}}\right) \ E[n,\boldsymbol{j}]$$

Current-Current interaction are related to effective masses (L=1 Landau parameter) $C^\tau = -C^{\boldsymbol{j}}$

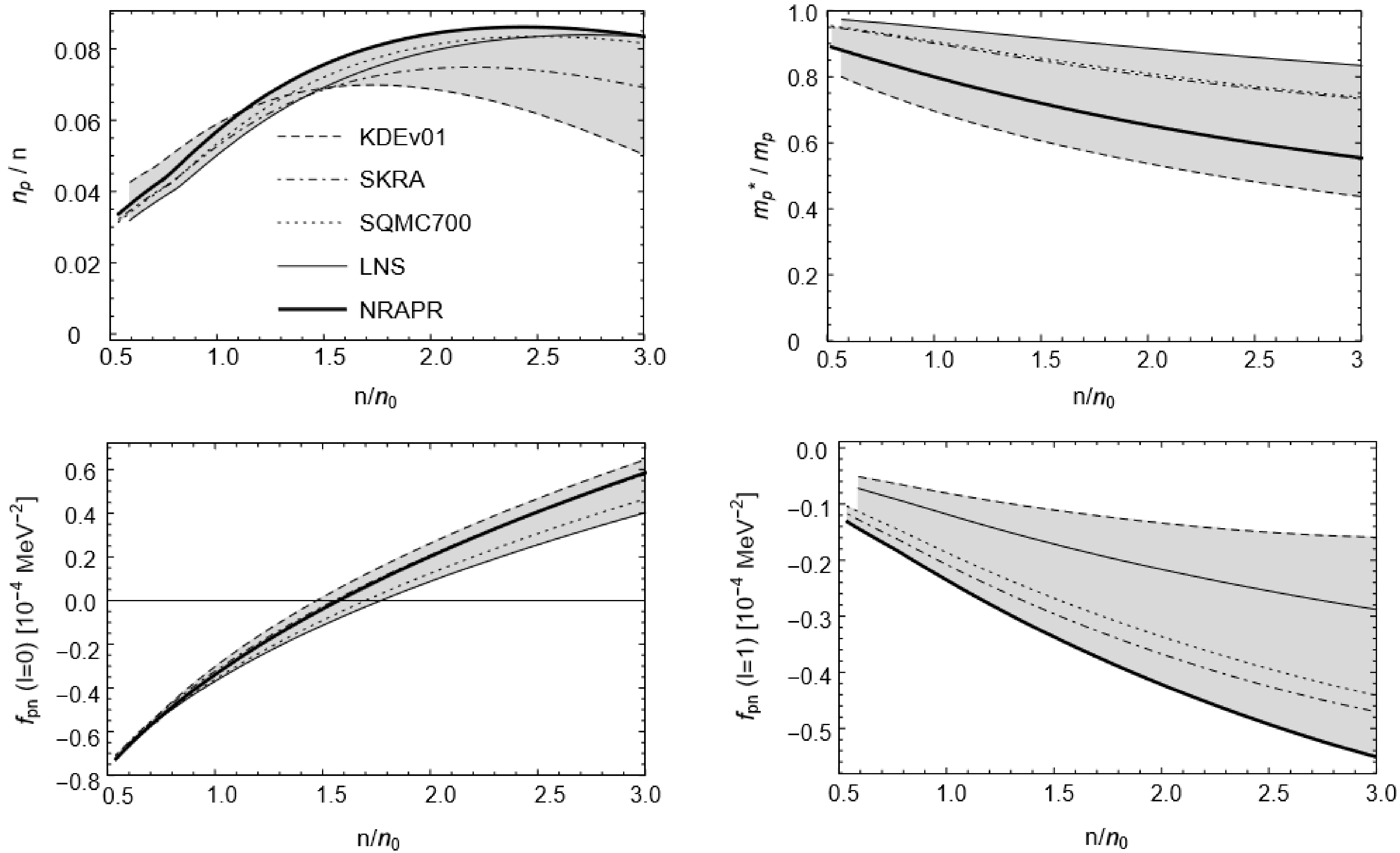
$$\frac{\hbar^2}{2m_a^*} = \frac{\delta E}{\delta \tau_a} = \frac{\hbar^2}{2m_a} + (C_0^\tau - C_1^\tau)n + 2C_1^\tau n_a$$

$C_{0,1}^{n,\tau,\boldsymbol{j}}$ related to Skyrme parameters. Here: NRAPR, SKRA, SQMC700, LNS, KDE0v1

[M. Dutra, O. Lourenço, J. S. Sá Martins, A. Delfino, J. R. Stone, P. D. Stevenson , PRC 85 ,035201]

→ **matching to relativistic field theory variables required**

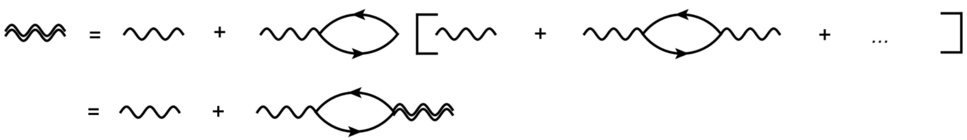
Strongly interacting Fermi Liquid (II)



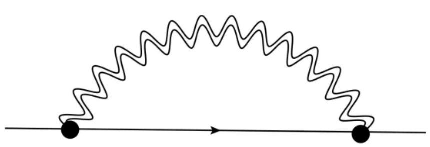
From field theory to transport

consider simplest case: single fermion species interacting electromagnetically (QED)

I) in-medium dispersion of photons



II) in-medium dispersion of fermions



"side quests"

- screening and damping
- collective modes

→ spectral function of photons

[S. Stetina, E. Rrapaj, S. Reddy
Phys. Rev. C 97, 045801]

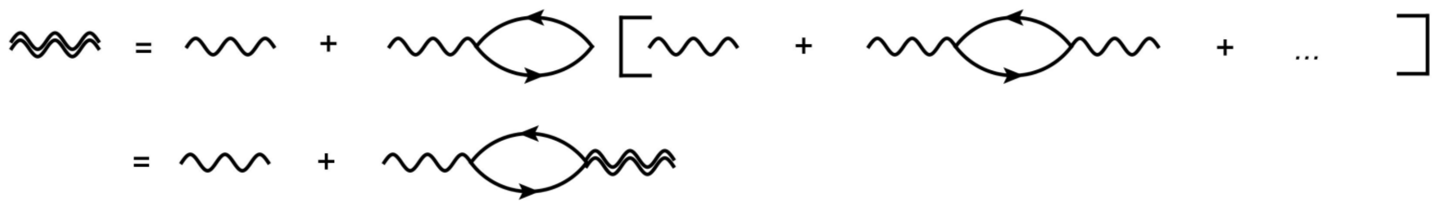
- spectral function of fermions in degenerate matter
- scattering rates, i.e., $\mathrm{Im}\,\Sigma\,(\mathrm{p}_0, \mathbf{p})$

[S. Stetina, in preparation]

in terms of entropy production rate $S' \propto \int d^3\boldsymbol{p}\,\mathrm{Im}\,\Sigma\,(\mathrm{p}_0, \mathbf{p})$

$$\kappa^{-1} = T^2 S' \,/\, j_H^2 \qquad \sigma^{-1} = T\, S' \,/ j_E^2 \qquad \eta^{-1} = 2T\, S' \,/\, \left(\Pi_{ij}\right)^2$$

Part I: Photon Spectrum in Dense Nuclear Matter



Photon spectrum (I): RPA

Relativistic one-loop resummation ("Random Phase Approximation" , RPA)

$$\widetilde{D}^{\mu\nu}(q) - D^{\mu\nu}(q) = \Pi^{\mu\nu}(q)$$

- Dressed photon propagator in ***Coulomb Gauge***:

$$\widetilde{D}^{\mu\nu}(q_0, q) = \frac{q^2}{\boldsymbol{q}^2}(\boldsymbol{q}^2 - \Pi_L)^{-1} P_L^{\mu\nu} + (q^2 - \Pi_\perp)^{-1} P_\perp^{\mu\nu}$$

- hard region: $q = (q_0,\ \boldsymbol{q}) \sim\ k_f$ soft region (medium effects): $q \sim e\, k_f$

→ Weak screening approximation:

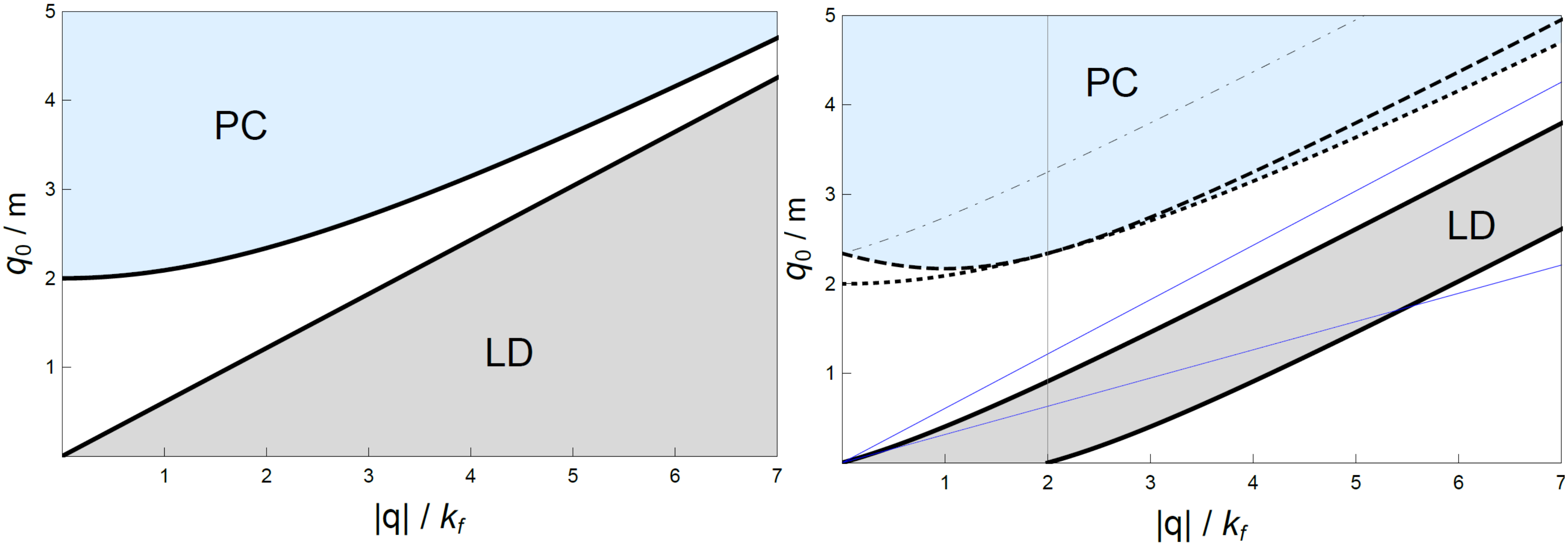
$$D_L \propto \frac{1}{q^2 - m_D^2}\ , \qquad D_\perp \propto \frac{1}{q^2 - i\left(\frac{q_0}{|\boldsymbol{q}|}\right) q_{\mathrm{f}}^2}\ , \qquad m_D^2 = \frac{4\alpha_f}{\pi}\mu k_f\ , \qquad q_{\mathrm{f}}^2 = \alpha_f k_f^2$$

Extensively used in the calculation of transport in NS:

[E. Flowers and N. Itoh, Astrophys. J.206, 218 (1976)] [P.S. Shternin, D.G. Yakovlev Phys.Rev.D78 (2008), 063006]
[P.S. Shternin, D.G. Yakovlev Phys.Rev.D75 (2007), 103004]

→ Hard dense loop (HDL) approximation $q \ll k_f$ (requires m $\ll k_f$)

Photon spectrum (II): damping $\Gamma_{L,\perp} \propto Im\, \Pi_{L,\perp}$



Regions where pair creation (PC) and Landau damping (LD , i.e., p-h creation) operate:

$\mu = 0$ (left) :

$q_0 = |\boldsymbol{q}|$ (LD)

$q_0 = \sqrt{|\boldsymbol{q}|^2 + 4\, m^2}$ (PC)

(right: **dotted**)

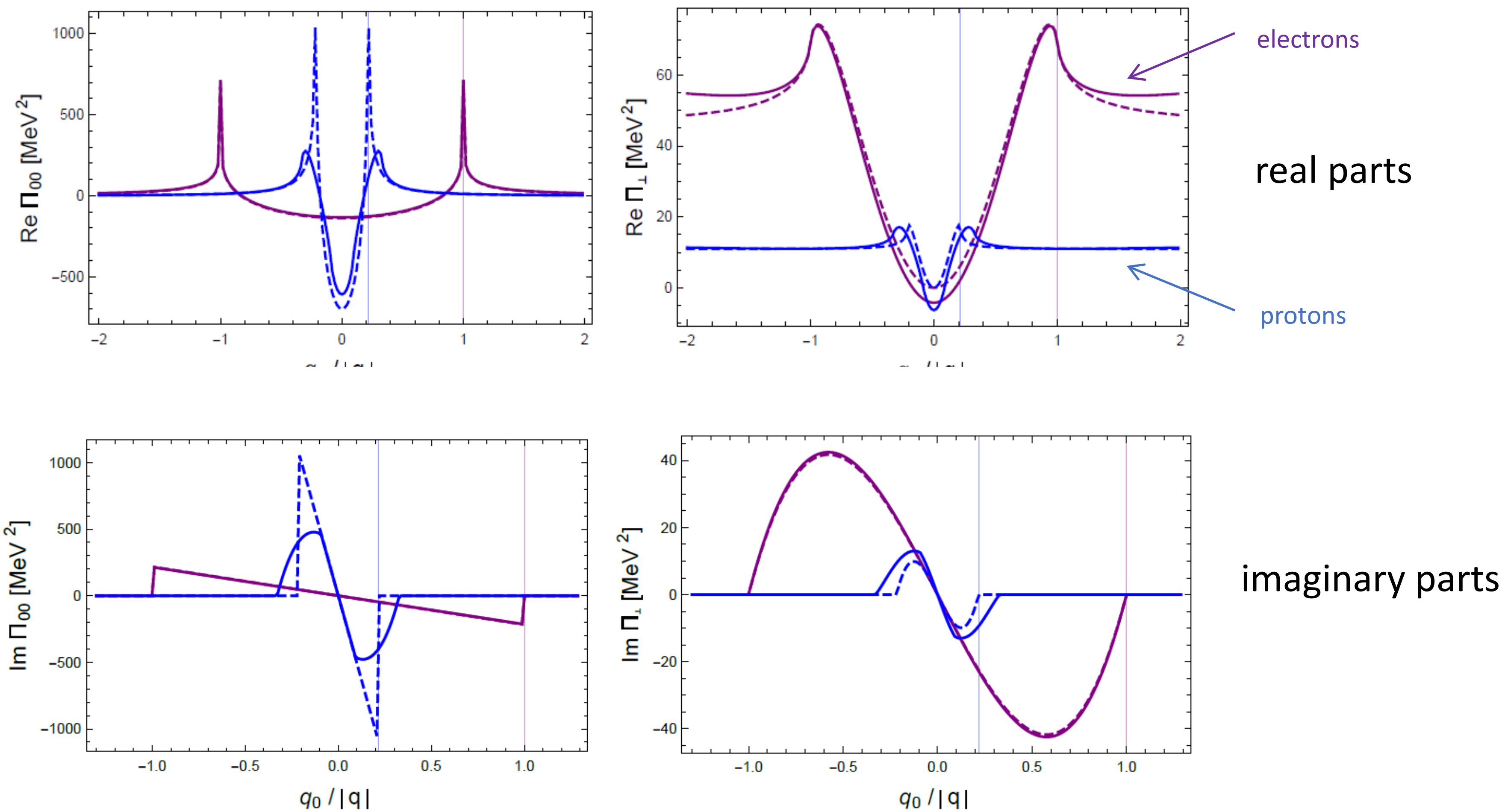
degenerate matter (right) :

$q_0 = -\mu + \sqrt{\mu^2 + |\boldsymbol{q}|^2 \pm 2k_f|\boldsymbol{q}|}$ (LD, **solid**)

$q_0 = +\mu + \sqrt{\mu^2 + |\boldsymbol{q}|^2 - 2k_f|\boldsymbol{q}|}$ (PC, $|\boldsymbol{q}| > 2k_f$, **dashed**)

Photon spectrum (III): real parts (full vs HDL)

hard dense loops: $q_0 \ll \mu, \quad |\boldsymbol{q}| \ll k_f$



at $\mathbf{n = n_0}$: $\mu_e \sim 120$ MeV $\quad \mu_p^* \sim 600$ MeV $\quad m_p^* \sim 575$ MeV $\quad |\boldsymbol{q}| \sim 0.2\,\mu$

collective modes

collective modes (poles of the resummed propagator)

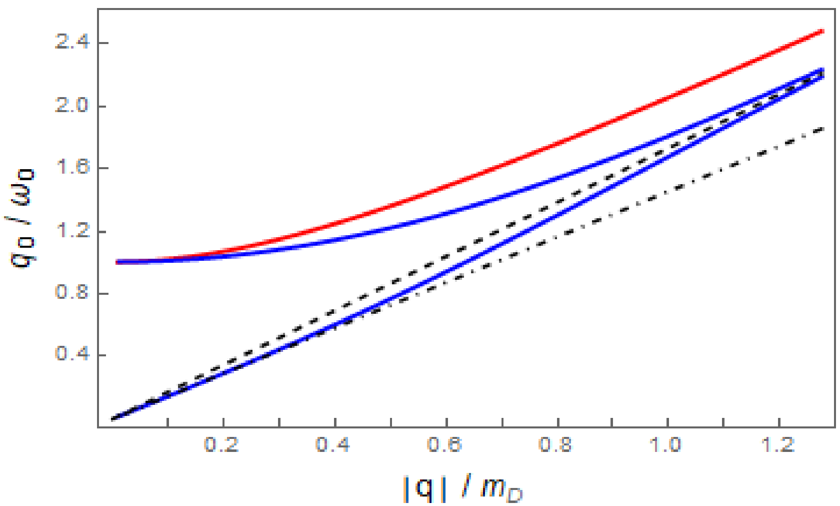
$\mathbf{q}^2 = Re\,\Pi_L(\omega_L, \boldsymbol{q})$,

plasmon (+ overdamped solution)

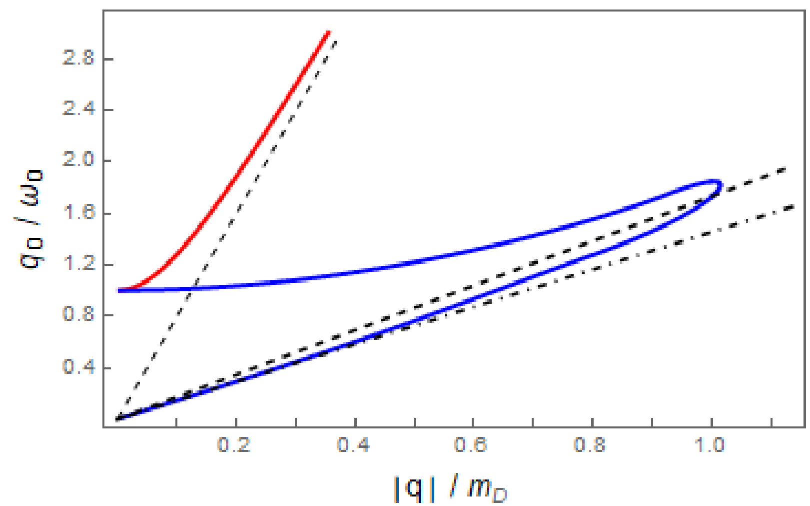
$\omega_\perp^2 = \mathbf{q}^2 + Re\,\Pi_\perp(\omega_\perp, \boldsymbol{q})$

photon

electron plasma



proton plasma



small **q**: $\omega_L^2 = \omega_0^2 + \frac{3}{5} v_f^2 \boldsymbol{q}^2$, $\quad \omega_< = c\, v_f |\boldsymbol{q}|$, $\quad \omega_\perp = \omega_0^2 + \left(1 + \frac{1}{5}\, v_f^2\right) \boldsymbol{q}^2$

“plasma frequency“: $\omega_0^2 = \frac{e^2}{3\pi^2} \frac{k_f^3}{\mu} = \frac{1}{3}\, v_f^2\, m_D^2$ slope of overdamped mode: $c \sim 0.83,$

collective modes: longitudinal damping

damping of modes

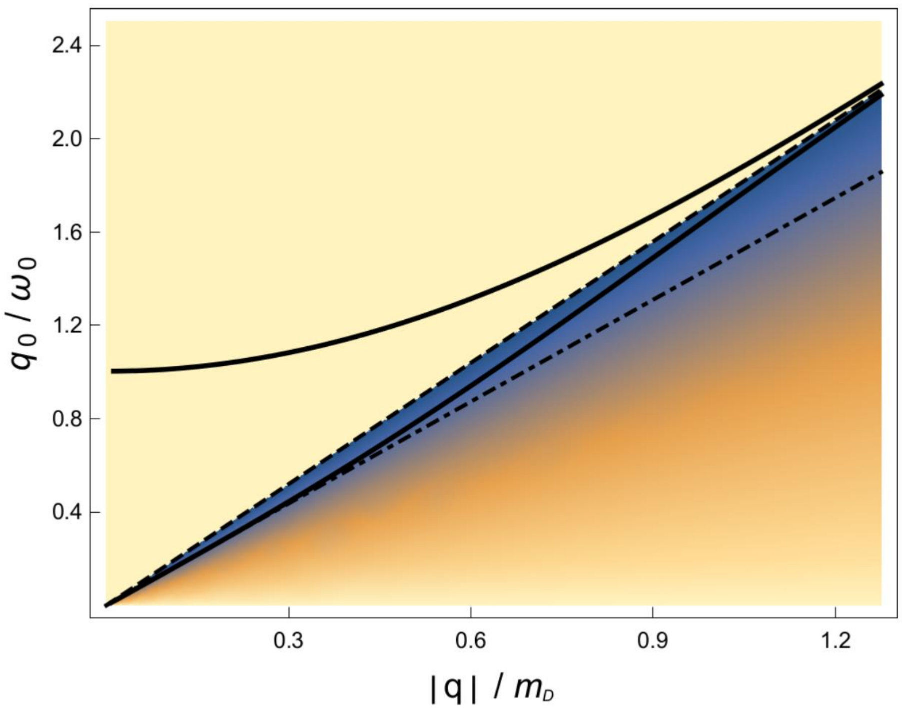
explains "thumb-like" shape of poles

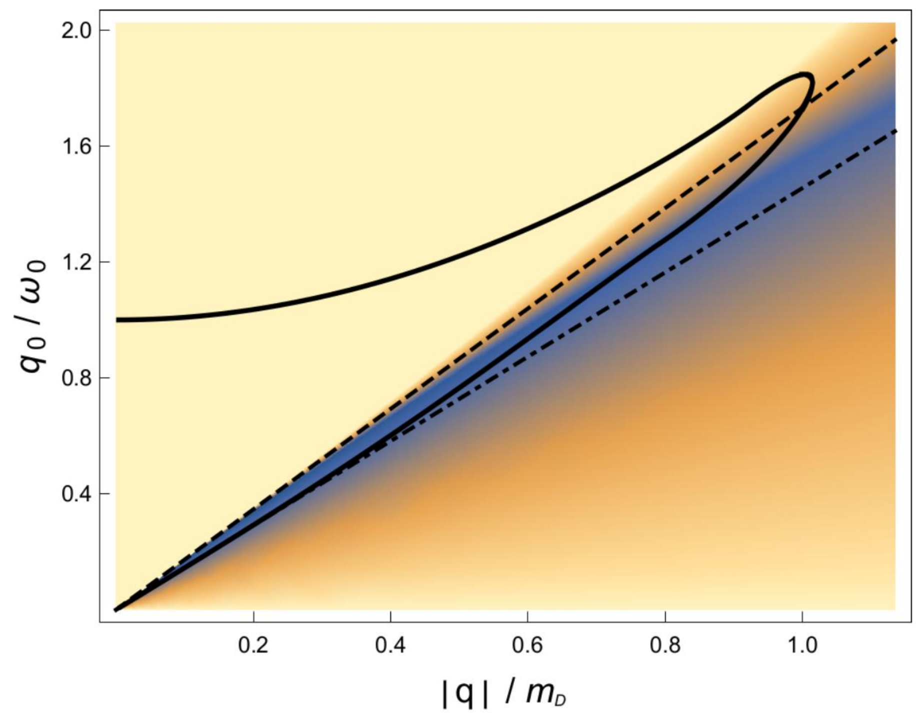
[L. McOrist, D.B. Melrose, J.I. Weise, arXiv: 0603227v1 [plasma-ph]]

[M. Baldo, C. Ducoin, PRC 79, 035801 (2009)]

HDL results: $(q_0, \boldsymbol{q}) \ll k_f, \mu$

$$Im\,\Pi_L = -\frac{\pi}{2} m_D^2 \frac{\mu}{k_f} \frac{q_0}{|\boldsymbol{q}|} \Theta(\mathrm{v_f}|\boldsymbol{q}| - \mathrm{q_0})$$



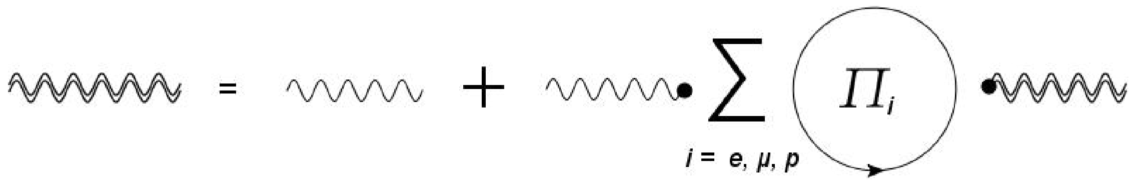


Photon spectrum: multi-component QED

Generalize to internal "flavour" space ($\boldsymbol{i \to e^-, \mu^-, p^+}$)

$$\Pi \to \mathrm{diag}\left(\Pi_e, \Pi_\mu, \Pi_p\right) \ , \qquad \gamma^\mu \to c^i \gamma^\mu \ , \qquad c^i = (1\ , 1, -1)$$

dressed photon propagator:



$$\widetilde{D}^{\mu\nu}(q_0, q) = \frac{q^2}{\boldsymbol{q}^2}(\boldsymbol{q}^2 - Tr\ [\Pi_L])^{-1}\ P_L^{\mu\nu} + (q^2 - Tr[\ \Pi_\perp])^{-1}\ P_\perp^{\mu\nu}$$

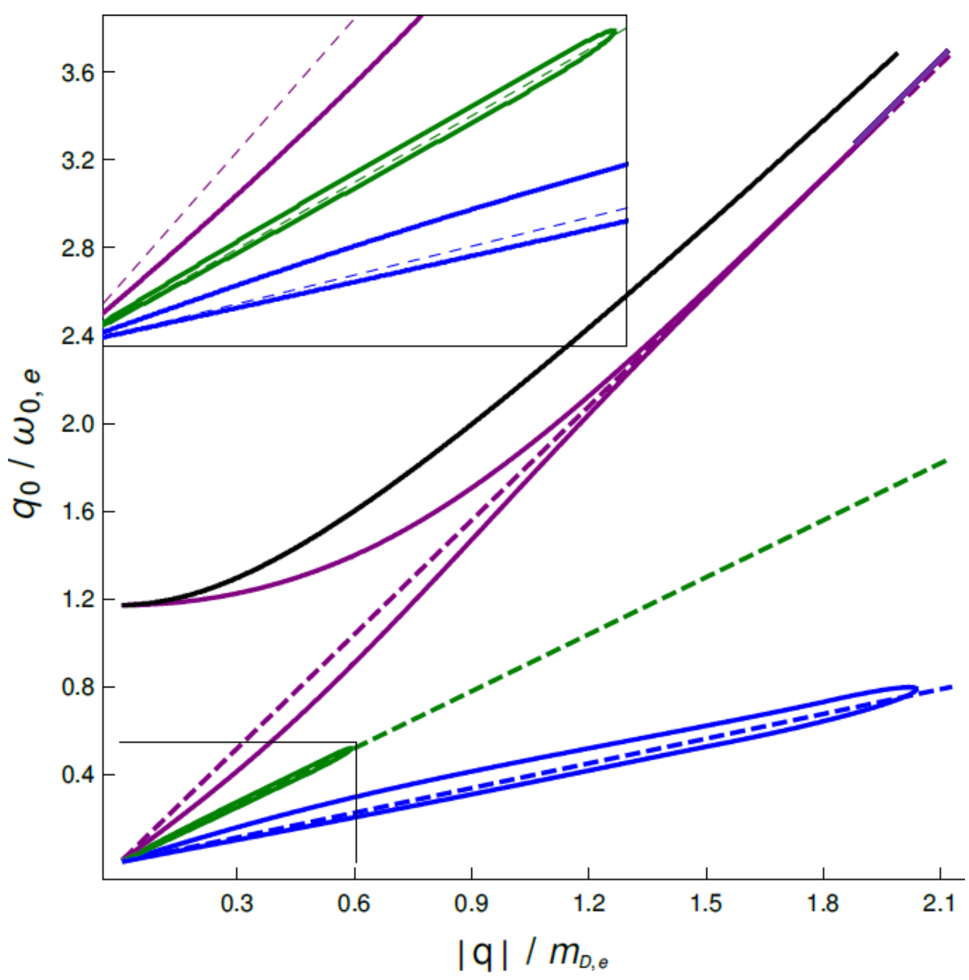
- **Protons are quasiparticles** (strongly interacting Fermi liquid) with effective masses m_p^*
- **Collective modes** are oscillation *in the densities* of these quasiparticles

RPA resummation in multi-component plasma is well established:

[C. Horowitz, K. Wehrberger, Nucl. Phys. A 531, 665 (1991)]

[S. Reddy, M. Prakash, J.M. Lattimer, J.A. Pons, PRC 59, 2888 (1999)]

Collective modes: multi-component QED



Spectrum (electrons, muons, protons)

- Three damped solutions (e, m, p)

  $\omega_{<,e}$, $\omega_{<,\mu}$, $\omega_{<,p}$

- Two Bohm-Staver sound modes (m, p)

  [D. Bohm, T. Staver , Phys. Rev. 84, 836 (1950)]

  u_μ , u_p

- One gapped (real) plasmon mode (e)

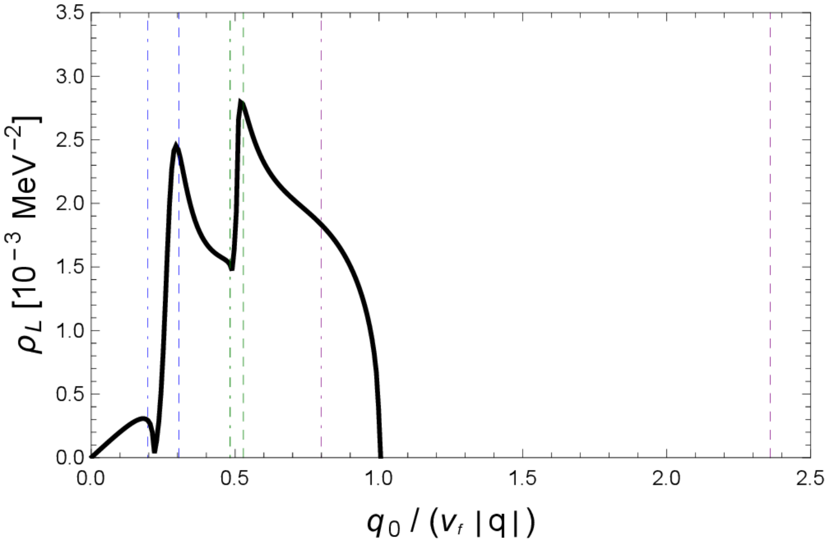
  ω_L

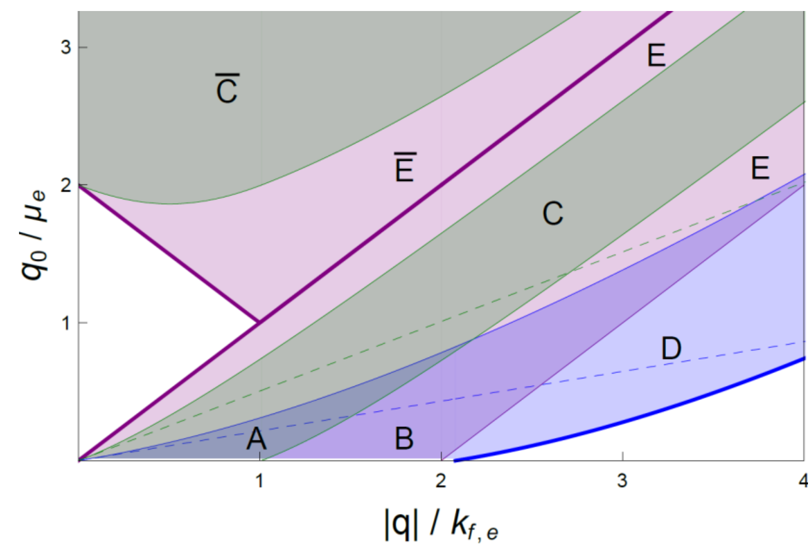
- transverse mode

  $\omega_\perp$

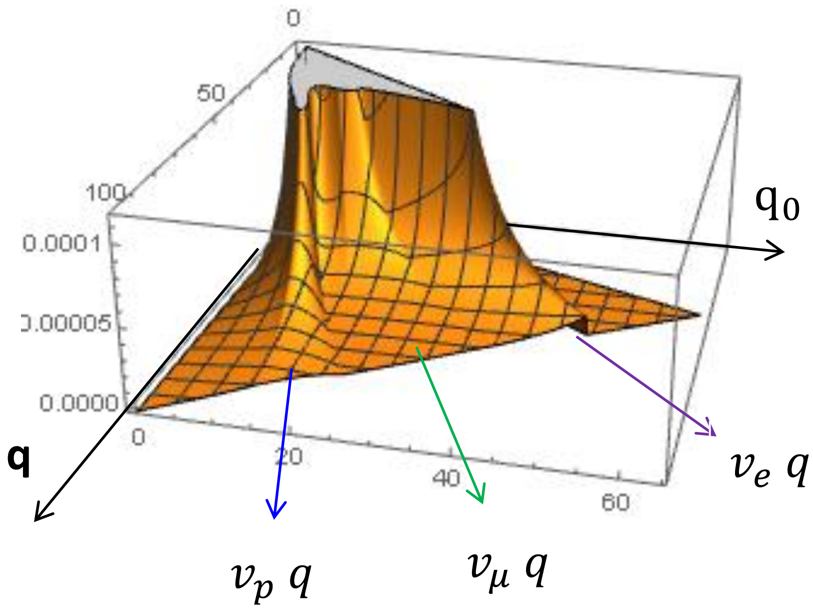
→ Light particles dynamically screen heavier ones.

Spectral functions: multi-component case





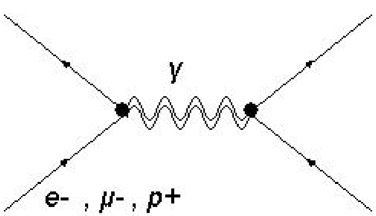
Landau damping in a e, m , p plasma



Static approx. :

$$\Pi_L(q_0 = 0) = \frac{e^2}{\pi^2}\mu k_f \ ,$$

$$\Pi_\perp(q_0 = 0\ ) = 0\ \ .$$



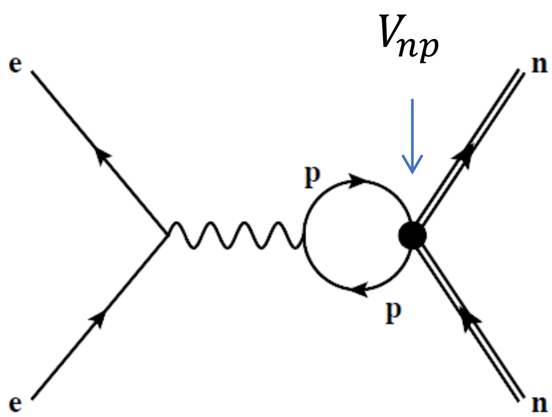
QED + strong interactions

What's the role of the neutrons within RPA?

- Coupling to photon tiny in free space

- BUT: Interactions induced by the polarizability of protons
  [B. Bertoni, S. Reddy, E. Rrapaj, Phys. Rev. C 91, 025806 (2015)]

- use *resummed* RPA polarization tensor for protons
  [S. Reddy, M. Prakash, J.M. Lattimer, J.A. Pons, PRC 59, 2888 (1999)]



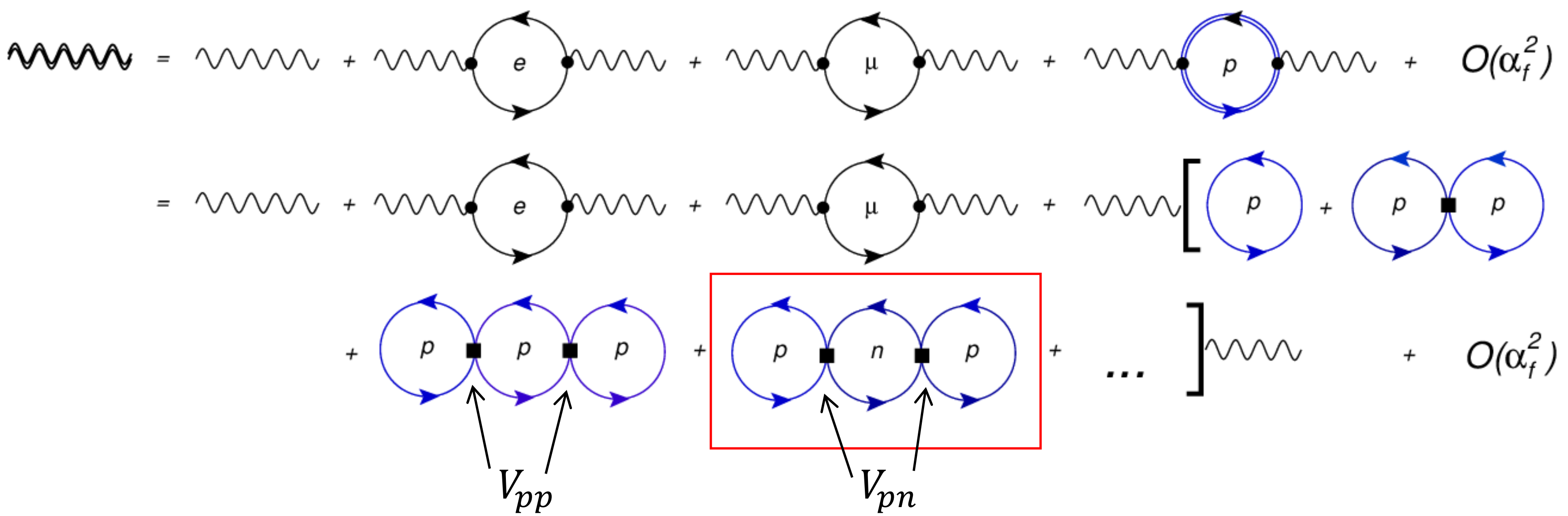
$$\widetilde{\Pi}_p = \frac{\Pi_p(1 - V_{nn}\Pi_n)}{1 - V_{nn}\Pi_n - V_{pp}\Pi_p + \left(V_{nn}V_{pp} - V_{np}^2\right)\Pi_n\Pi_p}$$

$$\widetilde{D}^{\mu\nu}(q_0, q) = \frac{q^2}{\boldsymbol{q}^2}\left(q^2 - \Pi_{e,L} - \Pi_{\mu,L} - \widetilde{\Pi}_{p,L}\right)^{-1} P_L^{\mu\nu} + \left(q^2 - \Pi_{e,\perp} - \Pi_{\mu,\perp} - \widetilde{\Pi}_{p,\perp}\right)^{-1} P_\perp^{\mu\nu}$$

- Quasiparticle properties and interaction potentials V_{ij} obtained from Landau energy functional based on Skyrme type interactions [N. Chamel, P. Haensel, PRC.73, 045802]

“Induced“ interactions

Nuclear interactions appear „nested“ inside electromagnetic ones



- V_{ab} are described by pointlike short-range interactions:

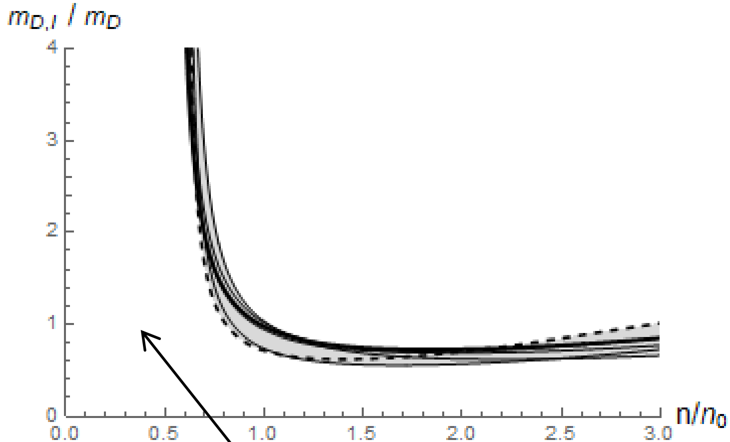
e μ p n

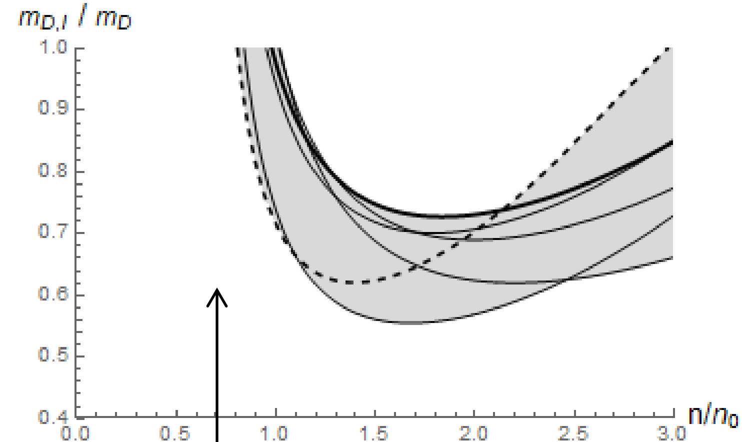
$$V^{\mu\nu}(q) = \frac{P_L^{\mu\nu} + P_\perp^{\mu\nu}}{q^2 + i\epsilon}\begin{pmatrix} 1 & 1 & -1 & 0 \\ 1 & 1 & -1 & 0 \\ -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \frac{q^2}{\boldsymbol{q}^2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & V_{pp} & V_{pn} \\ 0 & 0 & V_{np} & V_{nn} \end{pmatrix} g^{\mu 0} g^{\nu 0} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{V}_{pp} & \bar{V}_{pn} \\ 0 & 0 & \bar{V}_{np} & \bar{V}_{nn} \end{pmatrix} g^{\mu i} g^{\nu j}$$

density-density (l=0) current-current (l=1)

“induced” (strong) screening

TD definition: $\widetilde{\Pi}_{L,p}(q_0 = 0) = \left[\frac{\partial \mu_p(\mu_n)}{\partial n_p}\right]^{-1} = \frac{m_p^2\,(1+V_{nn}\,m_n^2)}{1+V_{nn}\,m_n^2+V_{pp}\,m_p^2+(V_{nn}V_{pp}-V_{np}^2)\,m_n^2\,m_p^2}$





homogeneous nuclear matter unstable

→ **induced interactions most pronounced at densities close to the crust-core boundary**

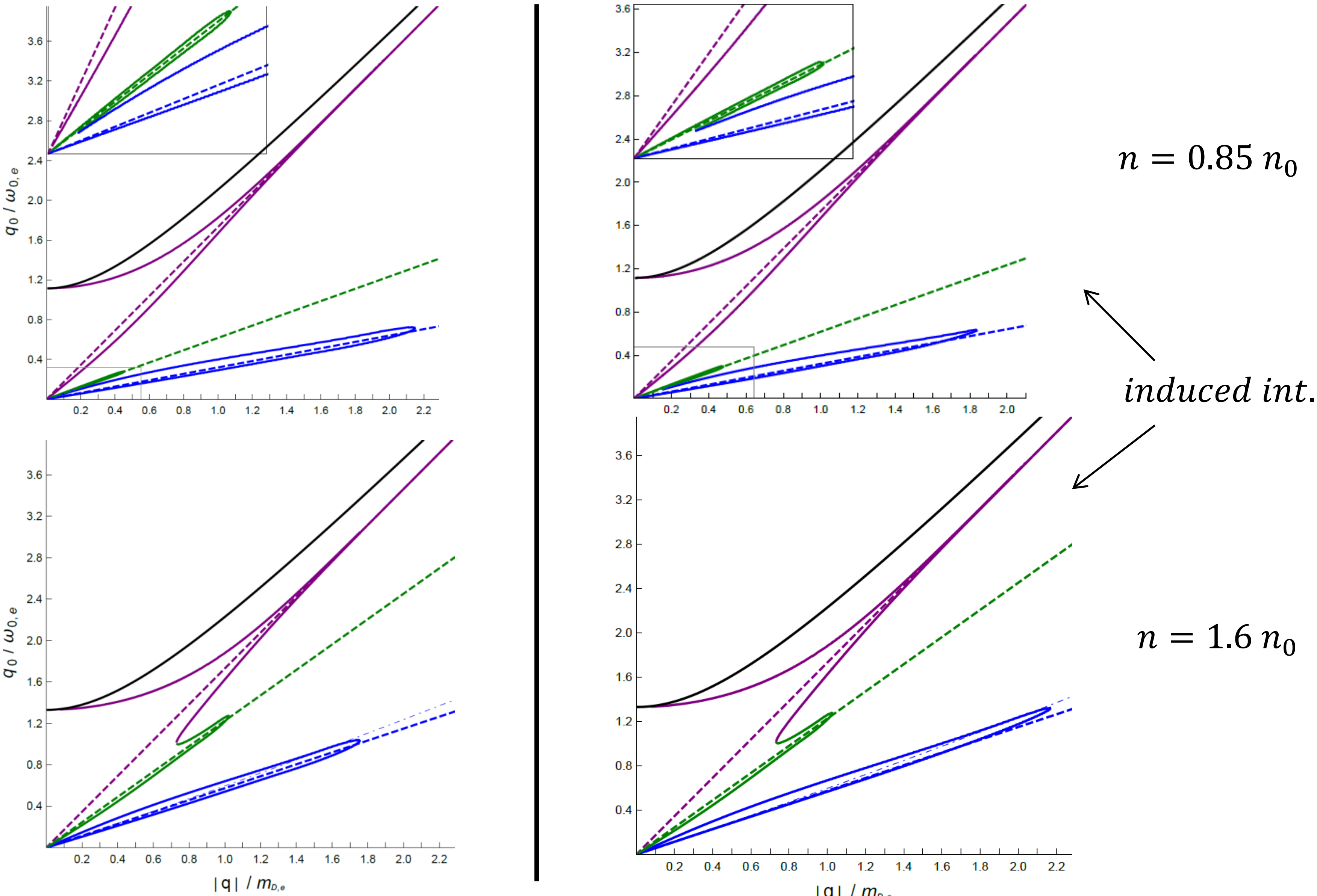
qualitative impact very robust, present in any Skyrme parameter set tested

→ changes to transverse spectrum are negligible since for protons $\Pi_\perp \ll \Pi_L$.

$$L_{\gamma-n} = e^2\, V_{np}\, (\bar{n}\,\gamma_\mu\, n)\; A_\nu\, (\Pi_{L,p}\, P_L^{\mu\nu} + \Pi_{\perp,p}\, P_\perp^{\mu\nu})$$

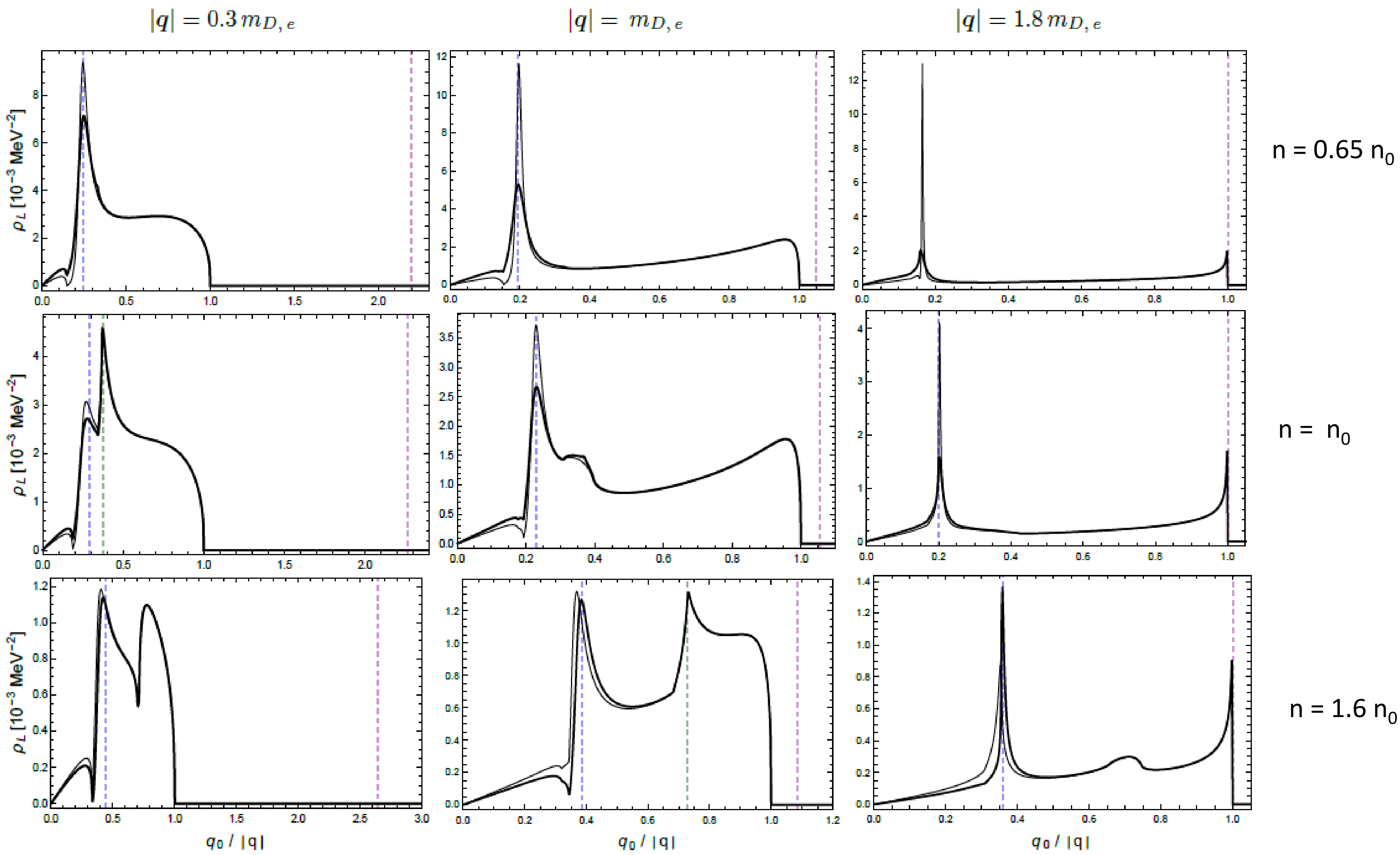
Collective modes: QED + strong int.

compare to: [M. Baldo, C. Ducoin, PRC 79, 035801 (2009)]

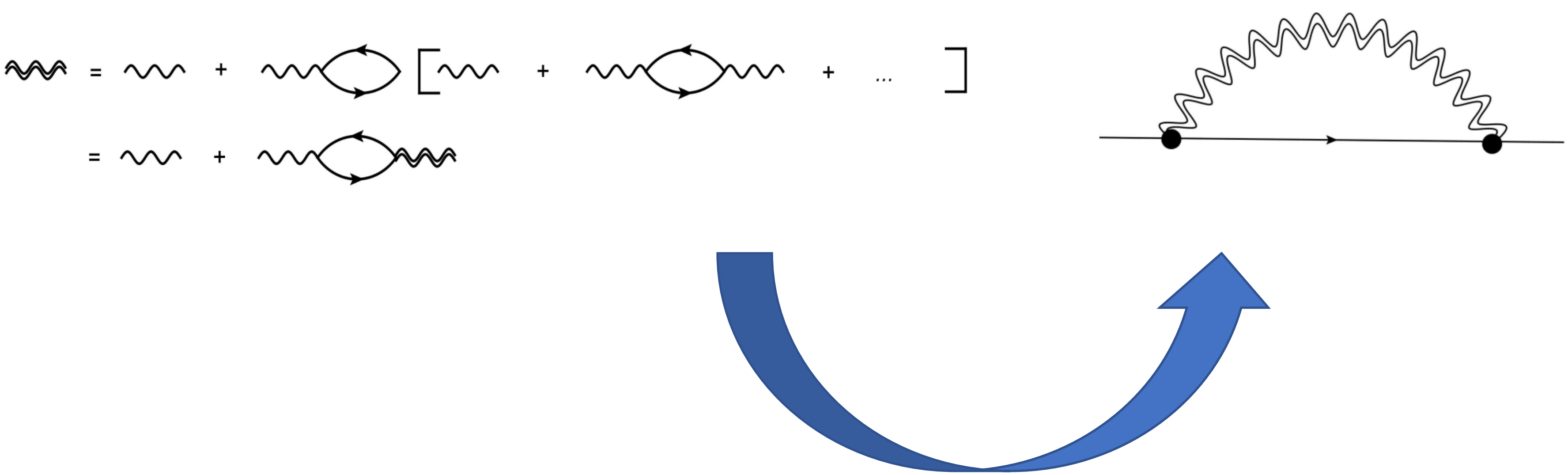


Spectral functions: QED + strong int.

compare to: [M. Baldo, C. Ducoin, PRC 79, 035801 (2009)]

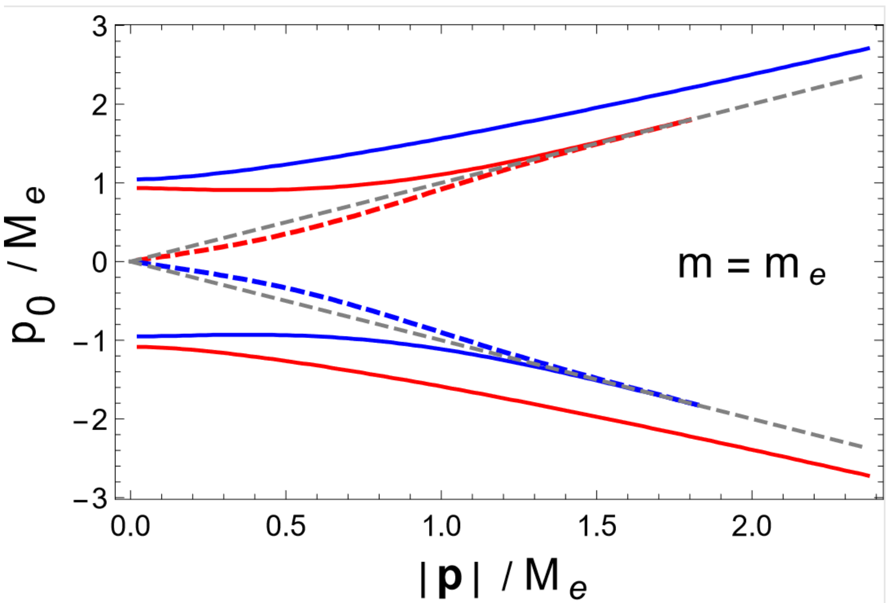


Part II: Electron (Muon) Damping in Dense Nuclear Matter



Scattering rates of fermions

fermion ($|\boldsymbol{p}| > k_f$) and hole ($|\boldsymbol{p}| < k_f$) dispersion relations in degenerate matter



soft fermionic excitations include

Λ^+: particles (or holes), anti-plasmino

Λ^- : anti-particle, plasmino

→ soft fermion spectrum more involved (mode mixing, non-perturbative effects)
[J. P. Blaizot, J.Y. Ollitrault, PRD 48, 3 1993]
[S. Stetina, in preparation]

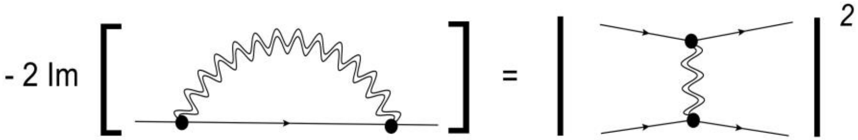
$$S(p) = [S_+ \Lambda_{+,\boldsymbol{p}} + S_- \Lambda_{-,\boldsymbol{p}}]\gamma_0 \qquad S_\pm = [p_0 \mp (\epsilon_{\boldsymbol{p}} - \Sigma_\pm)]^{-1} \qquad \Lambda^\pm = \frac{1}{2}[1 + \gamma_0 \frac{\boldsymbol{\gamma}\cdot\boldsymbol{p}+m}{\epsilon_{\boldsymbol{p}}}]$$

scattering close to the Fermi surface:

→ **fermions** $p \sim k_f$ are always *on-shell* and *undamped* at order α_f

→ **photon** is either hard (large angle) or soft (small) angel

damping rate of fermion, single species



optical theorem

$$\Gamma_{+} = \frac{1}{2}\, Tr\,[\Lambda_{+}\,\gamma_0\,\mathrm{Im}\,\Sigma_{\mathrm{R}}] = -\frac{1}{2p_0}\, Tr[(\gamma\cdot p + m)\,\mathrm{Im}\,\Sigma_R\,(p_0, \boldsymbol{p}\,)]\,, \qquad p_0 = \epsilon_{\boldsymbol{p}}$$

photon spectrum $\rho^{\mu\nu} = \rho_L\, g^{\mu 0} g^{\nu 0} + \rho_\perp\, P_\perp^{\mu\nu}$

→ *week screening* & *close to Fermi surface* $\epsilon_{\boldsymbol{p}} - \mu \ll m_D$, $\qquad u = q_0 \,/\, |\boldsymbol{q}|$

$$\Gamma_L \simeq \frac{e^2}{4\pi}\frac{m_D^2}{v_f^2}\int_0^{|\epsilon_{\boldsymbol{p}}-\mu|} du\, u \int_0^\infty d|\boldsymbol{q}|\, \frac{1}{(m_D^2 + \boldsymbol{q}^2)^2} = \frac{e^2}{32}\frac{1}{m_D\, v_f^2}\,(\epsilon_{\boldsymbol{p}} - \mu)^2$$

$$\Gamma_\perp \simeq \frac{e^2}{4\pi} m_D^2\, v_f^2 \int_0^{|\epsilon_{\boldsymbol{p}}-\mu|} du\, u \int_0^\infty d|\boldsymbol{q}|\,|\boldsymbol{q}|\, \frac{4\,\boldsymbol{q}^2}{16\,\boldsymbol{q}^6 + u^2\,\pi^2 m_D^2\, v_f^2} = \frac{e^2}{12\pi}\, v_f |\epsilon_{\boldsymbol{p}} - \mu|$$

compare to: [C. Manuel, Phys.Rev. D62 (2000) 076009]

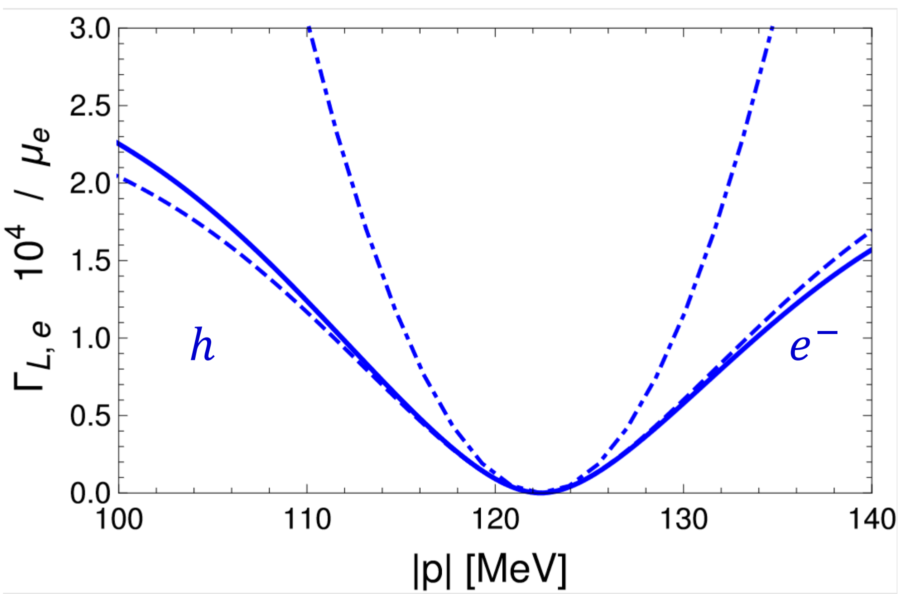
longitudinal and transverse damping

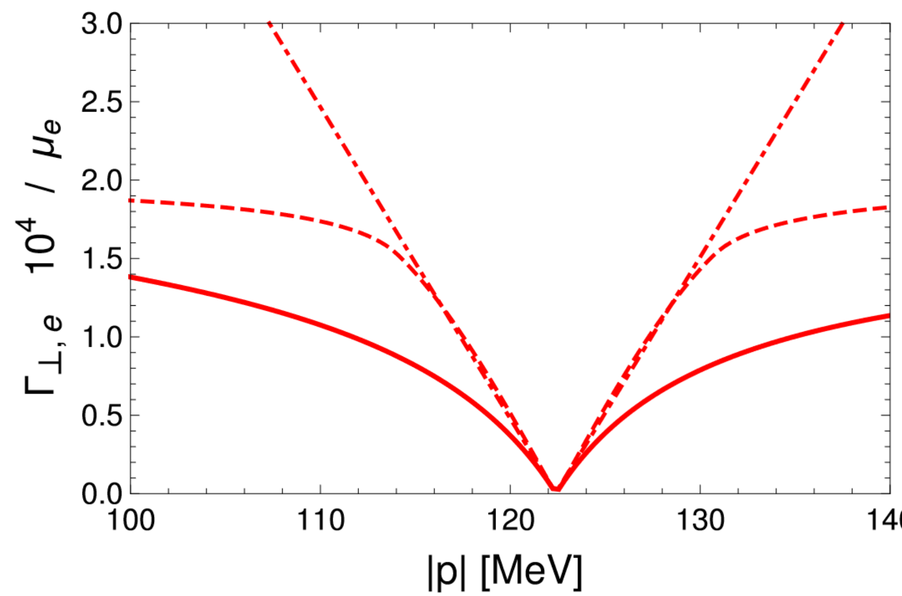
→ **nonrelativistic**: electric interactions dominate, magnetic interactions are down by $\left(\frac{v}{c}\right)^2$

→ **relativistic:** damping due to the exchange of plasmons and photons is equally important

[H. Heiselberg, G. Baym, C. J. Pethick, J. Popp, Nuc. Phys. A 544 (1992)]

electrons at n=n0





solid: full one-loop dashed: HDL dot-dashed: weak screening (static)

→ **HDL approximations work much better in the longitudinal channel!**

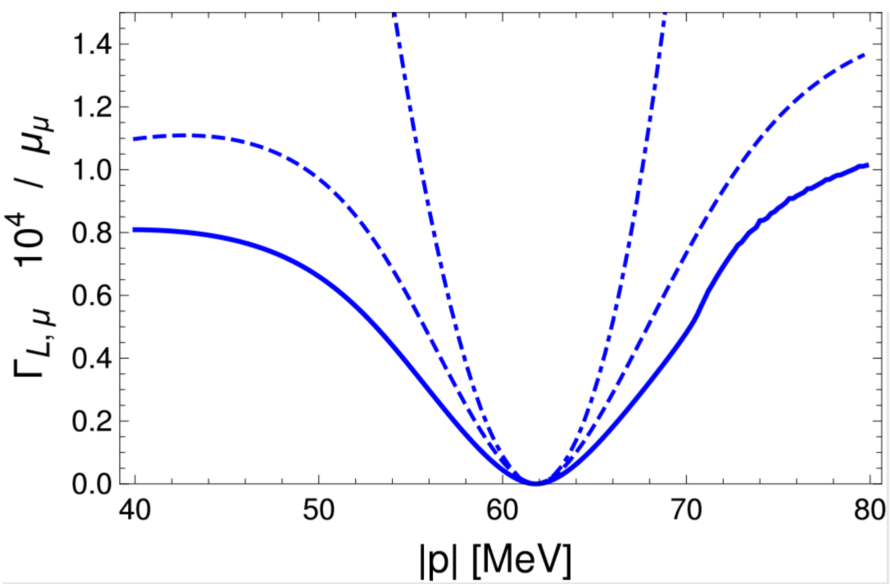
longitudinal and transverse damping

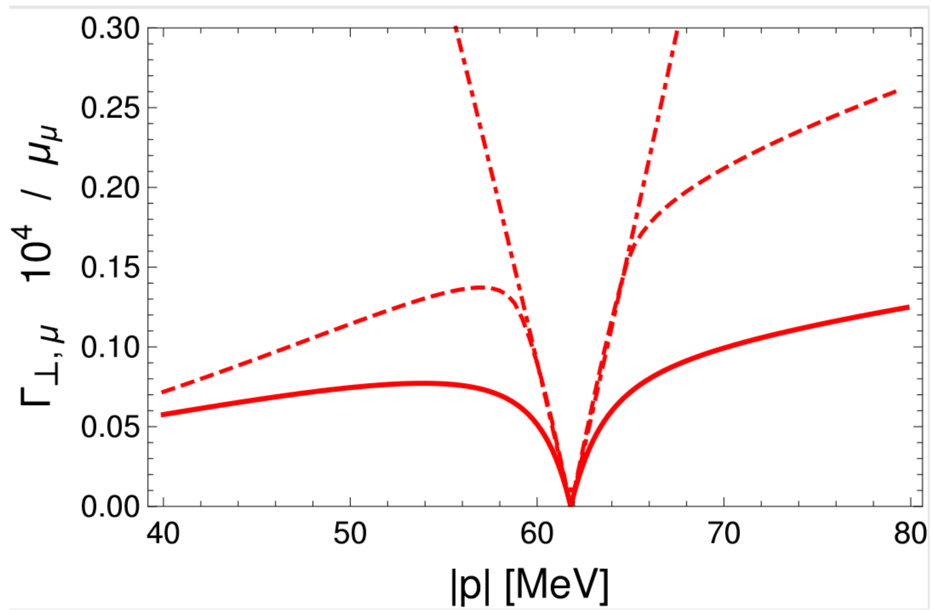
→ **nonrelativistic**: electric interactions dominate, magnetic interactions are down by $\left(\frac{v}{c}\right)^2$

→ **relativistic:** damping due to the exchange of plasmons and photons is equally important

[H. Heiselberg, G. Baym, C. J. Pethick, J. Popp, Nuc. Phys. A 544 (1992)]

muons at n=n0



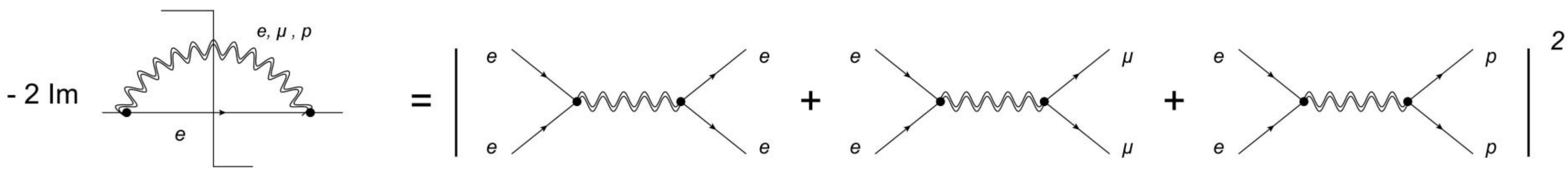


→ $|q| \ll k_f$ **hard to fulfill, HDL don't work really well in either channel**

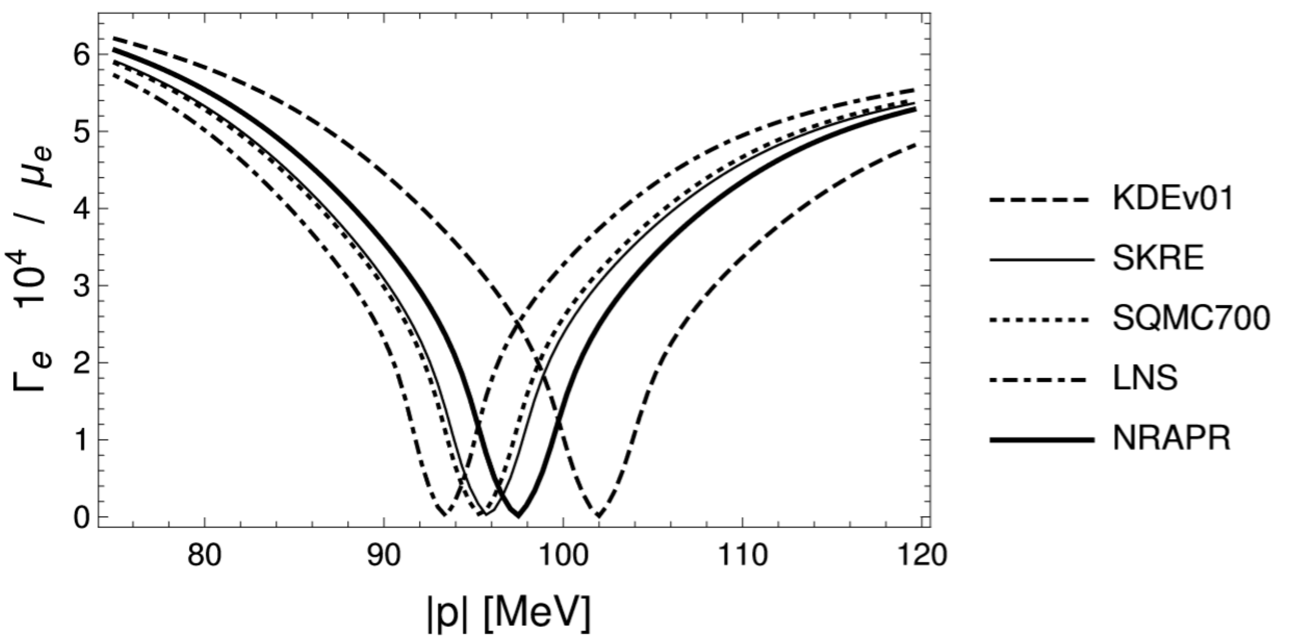
→ Γ_L **overtakes** $\Gamma_\perp$

damping rate of fermion, multiple species

energy loss of electrons due to collisions with other electrons, muons, and protons



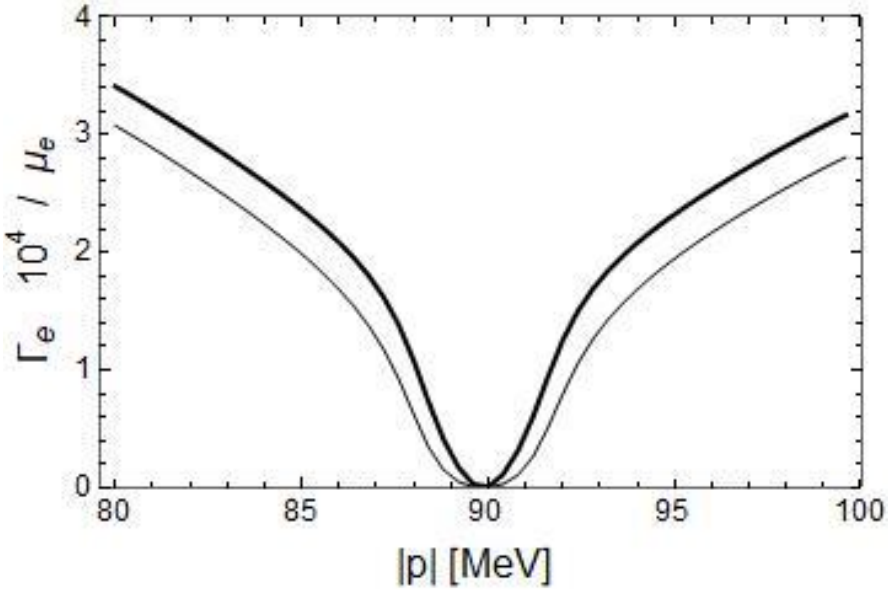
→ total screening mass: $M_D^2 = \sum_a m_{D,a}^2$ $\rho_L = \frac{1}{\pi} \frac{Tr[Im\,\Pi_{00}]}{(Tr[Re\Pi_{00}] - \boldsymbol{q}^2)^2 + (Tr[Im\,\Pi_{00}])^2}$



impact of induced interactions

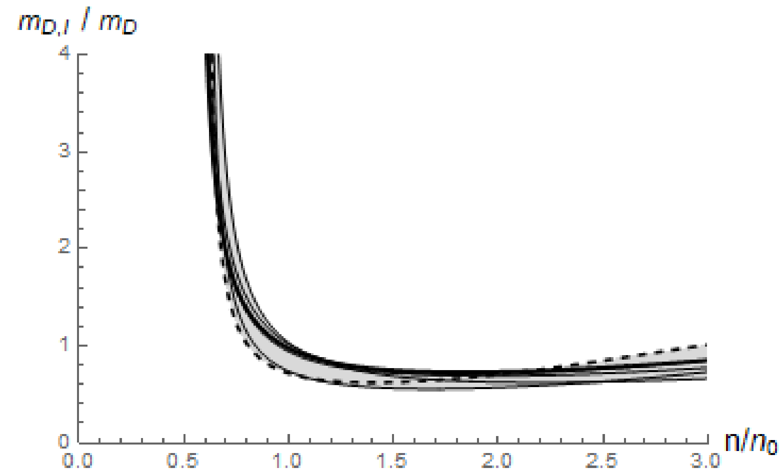
Γ_L modified near crust-core boundary

$\Gamma_\perp$ modifications negligible

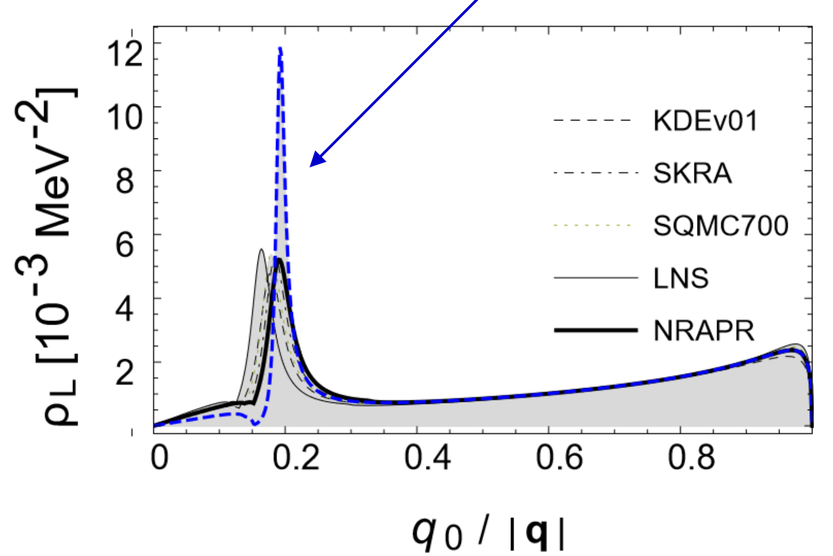


<u>caveat:</u>

dynamical screening effects should be incorporated in the nuclear sector



pure QED (NRAPR)



Outlook: transport (small energies q_0)

- Dynamical screening is important for the transverse damping rates
- Induced interactions are important for the longitudinal rates

Where to go from here:

- **Refine existing calculations of transport coefficients in neutron star cores.**

  [work in progress: E. Rrapaj, S. Reddy, S. Stetina]

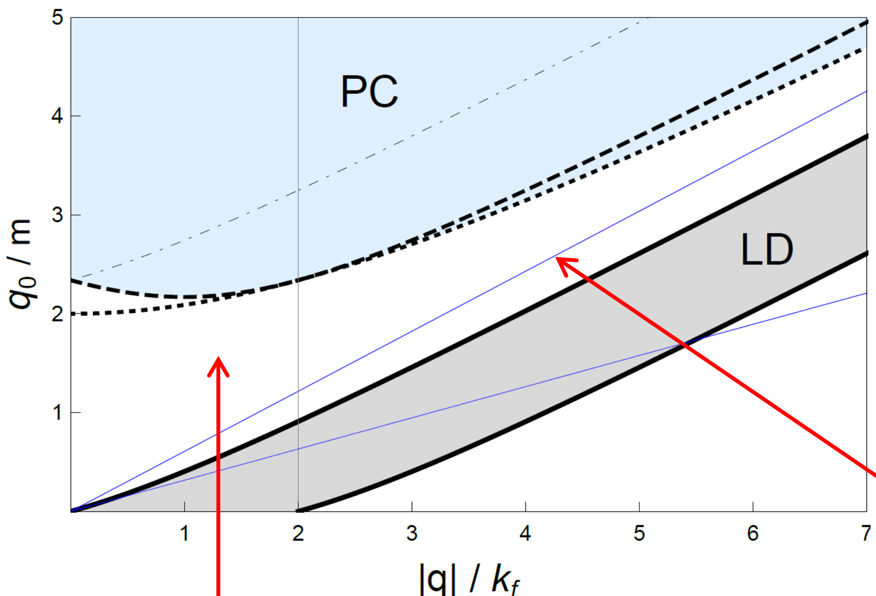
- **Improve the implementation of nuclear interaction potentials.**

- **if protons are superconducting: Meissner effect for (transverse) photon**
  → induced scattering dominates

  [B. Bertoni, S. Reddy, E. Rrapaj, Phys. Rev. C 91, 025806 (2015)]

Outlook (II): high energy spectrum, dark matter ?

- **beyond one-loop (RPA): what happens in diss. free region?**



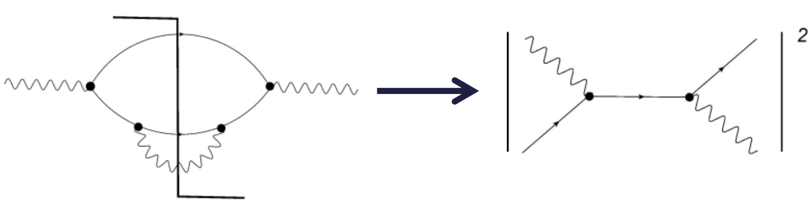
→ When one-loop results are kinemat. forbidden, two loop processes take over

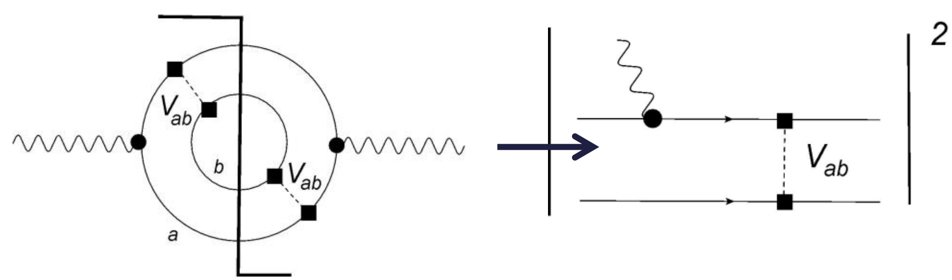
→ nucl. int. extracted from the measured nucleon-nucleon elastic cross section
[E. Rrapaj, S. Reddy, Phys.Rev. C94 (2016) no.4, 045805]

→ Study T dependence of processes (supernovae)

2 loop QED (e^4): Compton scatt.

2 loop QED + strong ($e^2 V_{ab}$): n-n Bremsstrahlung





Thank you!